A NEW ACCURATE EQUIVALENT NETWORK FOR STRIPLINE CIRCULATORS

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Summary

A new equivalent network accurately characterizing the immittance properties of a stripline circulator junction is proposed. The model is valid for circulators operating above as well as below resonance. Analytic expressions giving the element values of the equivalent network expressed in terms of the admittance properties of the junction are presented. The possibility of matching the new model for Chebyshev response using quarter wave transmission lines is shown. The usefulness of the new equivalent network is demonstrated by broadband matching of an experimental circulator.

I Introduction

Two of the most well-known equivalent network models for stripline circulator junctions are those of Bosma[1] and Fay-Comstock[2]. They consist of one or two parallel resonance circuits. The problem of matching these circuits has been treated by Anderson[3]. Another useful circuit model for stripline circulators was proposed by Helszajn[4]. It is composed of a quarter wave short circuited transmission line in parallel with a conductance, see Fig 1a. This model is attractive to use

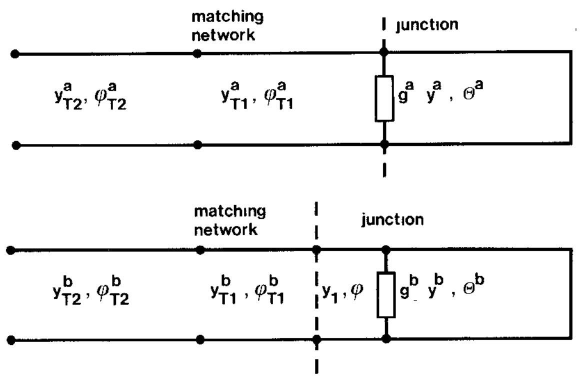

Fig 1 Equivalent network models including matching networks. a) Simple equivalent network. b) The new network.

in the design of stripline circulators because it can be matched to give Chebyshev response with quarter wave transmission line sections. For devices operating well below material resonance, the accuracy of this model is often very good. In other cases, however, the model does not possess the necessary properties to describe the real device.

Calculating the equivalent admittance at the plane of a circulator junction port using a multimode expansion of the fields the real part is normally found to increase with frequency in the vicinity of resonance. The slope is small for junctions operated below resonance but considerable in the above resonance case as shown, for instance, by Riblet[5]. The variation cannot be taken into account by the circuit in Fig 1a. This is one of the reasons why that model is often insufficient as an aid in the design of properly matched broadband circulators. In this paper we have improved the accuracy by using a slightly different network model.

II Equivalent network model

The variation of the conductance versus frequency is taken into account by including a short transmission line in the equivalent network model as indicated in Fig 1b. The proposed circuit has the potential properties which are required to model the known immittance performance of the component[5]. An advantage of our model is that the additional transmission line in the circuit can be looked upon as a part of the matching network used by Helszajn[4]. This fact indicates the possibility of using existing theories for matching with quarter wavelength transmission lines.

III Calculation of the network parameters

We now consider the input admittance y_{in} of the junction in Fig 1b. A straight-forward derivation gives the expressions in Eqs (1) and (2):

$$\mathrm{Re}\{y_{in}\} = \frac{g^b y_1^{\,2}(1+\tan^2\varphi)}{(y_1 + y^b\cot\theta\tan\varphi)^2 + (g^b)^2\tan^2\varphi} \qquad (1)$$

$$\mathrm{Im}\, y_{in} = \frac{y_1[\tan\varphi(y_1^{\,2} - (g^b)^2 - (y^b)^2\cot^2\theta) - y^b y_1\cot\theta(1-\tan^2\varphi)]}{(y_1 + y^b\cot\theta\tan\varphi)^2 + (g^b)^2\tan^2\varphi} \qquad (2)$$

For the determination of the parameters y_1, φ, g^b, and y^b we use (1) and (2) with $y_{in} = y_{inc}(f_c)$. Here y_{inc} is the input admittance of the junction port that we want to model at a frequency f_c, which can be chosen at, or in the vicinity of, the resonance frequency, f_0, of the junction. We also need the values of the derivatives of $\mathrm{Re}\{y_{inc}\}$ and $\mathrm{Im}\{y_{inc}\}$ at f_c for the calculation of the circuit parameters. This gives an equal number of equations and unknowns, from which it is possible to derive the following relations:

$$\varphi = \tan^{-1}\sqrt{\frac{J(H+J)}{1-JH}} \qquad (3)$$

$$g^b = \frac{2GJ(J+I)\ (1-JH)}{1+J^2} \quad (4) \qquad y_1 = \frac{g^b\tan\varphi}{J} \quad (5)$$

$$y^b = \frac{1}{y_1(1+\tan^2\varphi)}\cdot\left(\frac{G'f_c(y_1^2 + (g^b)^2\tan^2\varphi)^2}{\pi\, g^b y_1^{\,2}\tan\varphi} - \frac{2\varphi(y_1^2 - (g^b)^2)}{\pi\cos^2\varphi}\right) \qquad (6)$$

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where

$$H = B/G \qquad (7)$$

$$I = B'/G' \qquad (8)$$

$$J = \sqrt{1 + I^2} - I \qquad (9)$$

and

$$G = \mathrm{Re}(y_{inc}(f_c)) \qquad (10)$$

$$B = \mathrm{Im}(y_{inc}(f_c)) \qquad (11)$$

$$G' = d\mathrm{Re}(y_{inc})/df \Big|_{f = f_c} \qquad (12)$$

$$B' = d\mathrm{Im}(y_{inc})/df \Big|_{f = f_c} \qquad (13)$$

In this derivation the angle θ^b indicated in Fig 1b is chosen equal to $\pi/2$ at the frequency f_c.

IV Accuracy of the network model

In order to illustrate the quality of the new model we will use it to describe the circulation admittance of two junctions. One of them is operated above ferromagnetic resonance, while the other one is of below resonance type. They are designed to get the lowest order resonances at 0.88 GHz and 5.6 GHz, respectively. The circulation admittances of the junctions are calculated by a computer program including the three lowest order modes. From the computed characteristics of the junctions at the arbitrarily chosen center frequencies 0.85 GHz and 5.3 GHz, the parameters of the equivalent networks were calculated according to Eqs (3-6). The normalized input admittances of the equivalent networks are compared with the circulation admittances of the junctions in Fig 2a and b. The

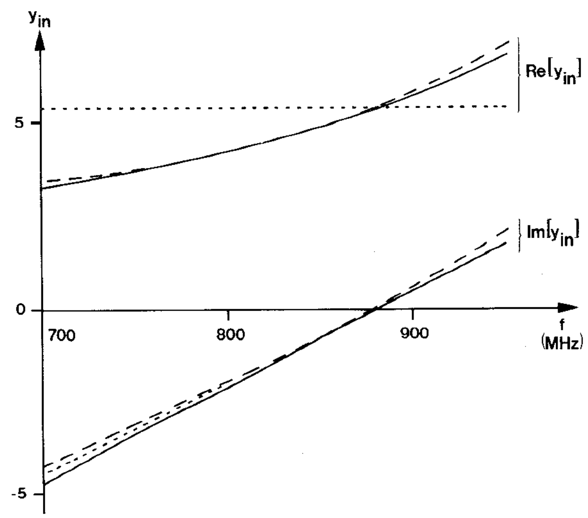
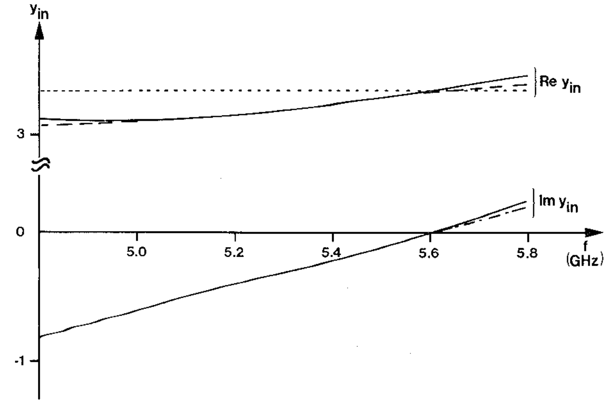


Fig 2 Calculated circulation admittance (——) compared with the input admittance of the new equivalent network (---) and that of Helszain's network (...)

a) Equivalent network calculated at 850 MHz.



b) Equivalent network calculated at 5.3 GHz.

admittance characteristics of the corresponding equivalent networks according to Helszajn are also given in the same figures. The accuracy of our model is in both cases very good.

The examples presented show, that the new equivalent network gives an accurate description of the admittance of junctions operated above as well as below resonance.

V Broadband matching with Chebyshev response

In Figs 1a and b Helszajn's and our model can be compared in their matched configuration. The network parameters in Fig 1a can be chosen in such a way that the return loss characteristics is of Chebyshev-type. The circuit in Fig 1b, however, can be made identical to that of Fig 1a if comparable parameters are chosen properly, i.e. $g^a = g^b$, $y^a = y^b$, $y^a_{T1} = y^b_{T1} = y^1$, $y^a_{T2} = y^b_{T2}$, $\theta^a = \theta^b$, $\varphi^a_{T1} = \varphi^b_{T1} + \varphi$ and $\varphi^a_{T2} = \varphi^b_{T2}$. This equality opens the possibility of using the new model together with the theories developed for the earlier one in obtaining Chebyshev response.

From Eqs. (4) - (6) it is obvious that if the conductance in Eq (10) and the susceptance in Eq (11) are multiplied by a factor k, corresponding to a change in the ferrite thickness, the network parameters g^b, y_1, and y^b will be multiplied by the same factor. In the single transformer case k should be chosen so that a transmission line with characteristic admittance $y^b_{T1} = ky_1$ is required for Chebyshev response. This condition gives:

$$k = (y^b/y_1 + 1)/g^b \qquad (14)$$

and the maximal VSWR, r

$$r = \frac{ky_1^2}{g^b} \qquad (15)$$

with a relative bandwidth of

$$2\delta_0 = 2\left\{1 - \frac{2}{\pi}\cos^{-1}\left(\sqrt{\frac{2(r-1)}{rkg^b - 1}}\right)\right\} \qquad (16)$$

By applying Eqs. (14) - (16) on the parameters for the C-band circulator in Fig 2b we get:

$k = 0.916 \qquad r = 1.065$

$2\delta_0 = 0.31 \qquad ky_1 = 1.786 \triangleq 28\Omega$

This result is in good agreement with experiments.

The bandwidth can be increased further if two transmission lines are used in the matching network. In this case k should be chosen so that transmission lines with characteristic admittances $y^b_{T1} = ky$, and y^b_{T2} give Chebyshev response. k and $2\delta_0$ are determined through the equations:

$$ky_1^2 = \frac{kg^b\left(1+\sqrt{kg^b}\right)^2 \sin^2\Theta\cos^2\Theta}{1-\left(\sqrt{kg^b}\cos^2\Theta - \sin^2\Theta\right)^2} \qquad (17)$$

$$ky^b\cot\Theta = \left[\left(\sqrt{kg^b}\sin^2\Theta - \cos^2\Theta\right)\left(\frac{1}{ky_1}+\frac{1}{y_1}\sqrt{\frac{g^b}{k}}\right) + \left(ky_1 + y_1\sqrt{\frac{k}{g^b}}\right)\left(\cos^2\Theta - \sqrt{\frac{1}{kg^b}}\sin^2\Theta\right)\right]\sin\Theta\cos\Theta \Bigg/ \left[\left(\frac{1}{ky_1}+\frac{1}{y_1}\sqrt{\frac{g^b}{k}}\right)^2\sin^2\Theta\cos^2\Theta + \left(\cos^2\Theta - \sqrt{\frac{1}{kg^b}}\sin^2\Theta\right)^2\right] \qquad (18)$$

where

$$\cos\Theta = \frac{\sqrt{3}}{2}\cos\left[\frac{\pi}{2}\left(1+\delta_0\right)\right] \qquad (19)$$

Since Eqs. (17) and (18) are nonlinear, k and $2\delta_0$ have to be solved by iterative methods.

The maximal VSWR, r, in the band is found by using Eq. (20) below.

$$\frac{r-1}{r+1} = \frac{\left[(Dkg^b - A - Bky^b\cot\Theta)^2 + (C - Bkg^b - Dky^b\cot\Theta)^2\right]^{1/2}}{\left[(Dkg^b + A + Bky^b\cot\Theta)^2 + (C + Bkg^b - Dky^b\cot\Theta)^2\right]^{1/2}} \qquad (20)$$

where

$$A = \cos^2\Theta - \sqrt{kg^b}\sin^2\Theta \qquad (21)$$

$$B = \left(\frac{1}{ky_1} + \frac{1}{y_1}\sqrt{\frac{g^b}{k}}\right)\sin\Theta\cos\Theta \qquad (22)$$

$$C = \left(ky_1 + y_1\sqrt{\frac{k}{g^b}}\right)\sin\Theta\cos\Theta \qquad (23)$$

$$D = \cos^2\Theta - \sqrt{\frac{1}{kg^b}}\sin^2\Theta \qquad (24)$$

and

$$\cos\Theta = \frac{1}{2}\cos\left[\frac{\pi}{2}\left(1+\delta_0\right)\right] \qquad (25)$$

The admittance y^b_{T2} is

$$y^b_{T2} = y_1\sqrt{\frac{k}{g^b}} \qquad (26)$$

VI Experiments

The necessity of a new network model became evident during the development of an above resonance circulator for broadband operation. Using the equivalent circuit of Fig 1a the desired bandwidth of 220 MHz centered around 900 MHz could not be achieved. Instead the circuit of Fig 1b with $y^b_{T1} + \varphi = \pi/2$ at the center frequency was used. The curves for return loss as measured in a network analyzer and theoretically calculated are shown in Fig 3. The result shows that

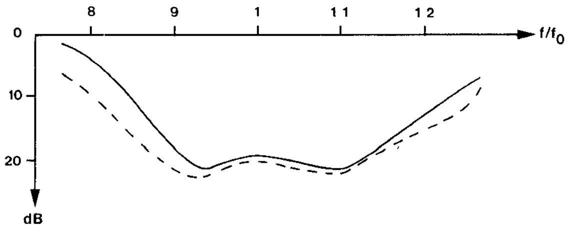


Fig 3 Return loss of a UHF-circulator operation above resonance measured (---) and theoretical (——), neglecting internal loss.

the new equivalent circuit is useful as an aid in the design of circulators also in the case of above resonance operation[5].

VII Conclusions

The new equivalent network model gives a good description of the input admittance of stripline circulators. By permitting a change in the admittance level earlier developed theories for Chebyshev matching can be applied. An implementation of the new model in the construction of an experimental circulator has demonstrated the usefulness of the new circuit mode.

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